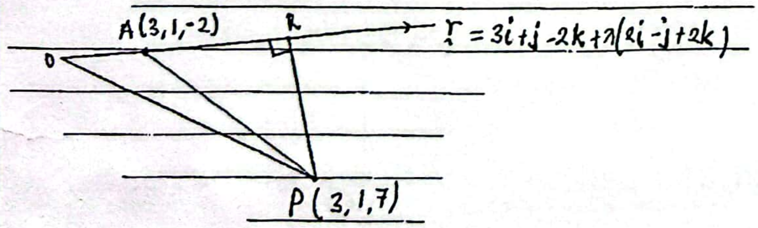
ODEKE. L

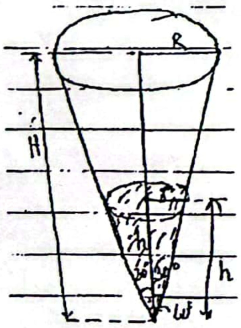
WAKISSHA JOINT MOCK EXAMINATIONS
MARKING GUIDE
Uganda Advanced Certificate of Education
UACE August 2023
MATHEMATICS P425/1

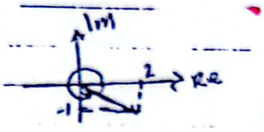
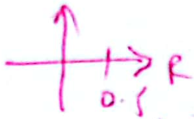
1.	$(a+b)^3 = a^2+3a^2b+3ab^2+b^3$ $(a+b)^3 = a^3+b^3+3ab(a+b)$ $(a+b)^3 = 6ab(a+b)+3ab(a+b)$ $(a+b)^3 = 9ab(a+b)$ $(a+b)^2 = 9ab$ $\sqrt{\frac{(a+b)^2}{9}} = \sqrt{ab}$ $\frac{a+b}{3} = a^{\frac{1}{2}}b^{\frac{1}{2}}$	B_1 stating the identity B_1 substituting in identity B_1 simplifying
	$\log\frac{a+b}{3} = \log(ab)^{\frac{1}{2}}$ $\log\frac{a+b}{3} = \frac{1}{2}\log(ab)$ $\log\left(\frac{a+b}{3}\right) = \frac{1}{2}(\log a+\log b)$	B_1 Applying logarithm B_1 Drawing conclusion
		05
2.	Let $y = \text{cosec}^{-1}(x)$ $\text{Cosec}y = x$ $-\text{Cosec}y\cot y\frac{dy}{dx} = 1$ $\frac{dy}{dx} = \frac{1}{\text{cosec}y\cot y}$ but $\cot y = \sqrt{(\text{cosec}^2y-1)}$ $\frac{dy}{dx} = \frac{-1}{(\cos cy)\left(\sqrt{(\text{cosec}^2y-1)}\right)}$ $\frac{dy}{dx} = \frac{-1}{x\left(\sqrt{(x^2-1)}\right)}$ $\therefore \frac{d}{dx}(\text{cosec}^{-1}(x)) = \frac{-1}{x\sqrt{(x^2-1)}}$ $\text{cosec}\,y = x$ $\frac{1}{\sin y} = x$ $\frac{d}{dx}(\sin y) = \frac{d}{dx}\left(\frac{1}{x}\right)$ $\cos y\frac{dy}{dx} = \frac{-1}{x^2}$ $\frac{dy}{dx} = \frac{-1}{x^2\cos y}$ but $\cos y = \sqrt{1-\sin^2 y}$ $\frac{dy}{dx} = \frac{-1}{x^2\sqrt{1-x^2}}$ $\frac{-1}{x\sqrt{(x^2-1)}}$	M_1 A_1 for differentiating and correct derivatives B_1 for making $\frac{dy}{dx}$ a subject. A1 M_1 for replacing coty A_1 for correct derivative A_1 B1
		05

3.	$\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{1+\tan x}dx$ $\frac{d}{dx}(1+\tan x)$ Let u = tan x du = $\sec^2 x\,dx$ $\frac{\sec^2 x}{1+\tan x}$ $dx = \frac{1}{\sec^2 x}du$ $Sm(1+\tan x)dx = \int\left(\frac{\sec^2 x}{1+\tan x}\right)dx$ $\begin{array}{\|c\|c\|}\hline x & u \\ \hline 0 & 0 \\ \hline \frac{\pi}{4} & 1 \\ \hline\end{array}$ $\int_0^{\pi/4} \frac{\sec^2 x}{1+\tan x}dx$ $\ln(1+\tan x)$ $\ln 2 - \ln 1$ $\ln 2 = 0.6931$	B_1 denvative of tan x B_1 change limits
	$\int_0^1 \frac{\sec^2}{1+u}\cdot\frac{1}{\sec^2 x}du$ $\int_0^1 \frac{1}{1+u}du$ $= [\ln(1+u)]_0^1$ $= = \ln 2 - \ln 1$ $= \ln 2$ $= 0.6931$	M_1 substituting u for tan x M_1 integration A_1 CAO
		05
4.	$\cos x + \sin 2x = 0$ $\cos x + 2\sin x\cos x = 0$ $\cos x(1+2\sin x) = 0$	B_1 (for expanding sin $2x$) M_1 (for factorizing)
	$\cos x = 0$ $x = \cos^{-1(0)}$ $90^0, 270^0$ $= \frac{\pi}{2}$ and $\frac{3}{2}\pi$	M_1 (for reading angle) A_1 (for angles in radians)
	OR	
	$\sin x = \frac{-1}{2}$ $x = \sin -1\left(\frac{-1}{2}\right)$ $= 210^0, 330^0$ $\pi/2, 7\pi/6, 3\pi/2, 11\pi/6$ $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$	$\overset{A}{B}_1$ (for angles in radius)
5.	Comparing $x^2+y^2-4y-5=0$ with $x^2+y^2+2gx+2fy+c=0$ $g = 0$, $f = {}^-2$ Centre (0~~, 2)~~ (2, 0) Radius $r_1 = \sqrt{g^2+f^2-c}$ $= \sqrt{(0^2+(-2)^2-{}^-5)} = \sqrt{9} = 3$	B_1 Finding the radius

	Comparing $x^2+y^2-8x+12y+1=0$ with $x^2+y^2+2gx+2+y+c=0$ $= g = -4, \quad f = 1 \quad e = 1$ centre (4,-1) Radius $r_2 = \sqrt{(-4)^2+1^2-1} = 4$	B$_1$ Finding the radius
	$c_1c_2 = \sqrt{(0-4)^2+(2-^-1)^2} = \sqrt{16+9} = 5$ $\overline{c_1c_2}^2 = 25$ $C_1C_2 = \sqrt{(4-2)^2+(-1-0)^2} = \sqrt5$ $r_1^2+r_2^2 = 3^2+4^2 = 25$ For two circles to be Orthogonal to each other $r_1^2+r_2^2 = \overline{c_1c_2}^2$ not $\therefore$ The circles are orthogonal.	M$_1$ finding the $\overline{c_1c_2}^2$ M$_1$Funding sum r_1^2 and r_2^2 A$_1$ drawing conclusion
6.	$\sqrt{\left(\frac{1+3x}{1-3x}\right)} = \sqrt{\frac{(1+3x)(1+3x)}{(1-3x)(1+3x)}} = (1+3x)(1-9x^2)^{\frac{-1}{2}}$ $(1-9x^2)^{\frac{-1}{2}} = 1+\frac{-1}{2}(-9x^2)+....$ $\cancel{1+}1+\frac{9}{2}x^2$ $(1+3x)\left(1+\frac{9x^2}{2}\right) = 1+3x+\frac{9x^2}{2}+\frac{27}{2}x^3$ $\sqrt{\left(\frac{1+3x}{1-3x}\right)} = 1+3x+\frac{9x^2}{2}+\frac{27}{2}\cancel{x^3}3$ $\left(\frac{1+\frac{3}{7}}{1-\frac{3}{7}}\right)^{\frac{1}{2}} = 1+3(\frac{1}{7})+\frac{9}{2}(\frac{1}{7})^2+\frac{27}{2}(\frac{1}{7})^3$ $\sqrt{10} = 2(1+0.428571428+0.091836734+0.0393586) = 3.12$	B$_1$ Simplifying B$_1$ for expanding $(1-9x^2)^{\frac{-1}{2}}$ B$_1$ for correct expansion M$_1$ for substitution A$_1$ (2dps)
		05
7.	 $\underset{\sim}{AP} = \begin{pmatrix}3\\1\\7\end{pmatrix} - \begin{pmatrix}3\\1\\-2\end{pmatrix} = \begin{pmatrix}0\\0\\9\end{pmatrix}$ $\underset{\sim}{U} = \begin{pmatrix}2\\^-1\\2\end{pmatrix}$	B$_1$ (for AP)

14, 14

	$\underset{\sim}{AP}\wedge\underset{\sim}{U}=\begin{vmatrix} i & j & k\\ 0 & 0 & 9\\ 2 & -1 & 12\end{vmatrix}$	M_1 for crossing
	i (0+9) – j(0-18)+k(0-0) $9\underset{\sim}{i}\ +\ 18\underset{\sim}{j}$	A1 for normal
	$\lvert\underset{\sim}{AP}\wedge\underset{\sim}{U}\rvert=\sqrt{9^2+18^2}=\sqrt{405}$ $\lvert\underset{\sim}{U}\rvert=\sqrt{2^2+(-1)^2+2^2}=3$	M1 for magnitudes of both AP ∧ U and U
	$\sqrt{45} =$ $D=\lvert\underset{\sim}{PR}\rvert\sqrt{\frac{405}{\cancel{3}\,9}}=\underline{6.7082\text{ units}}$	A1 for distance from P to the line.
		05
	OR	
	$\underset{\sim}{PR}=\left[\begin{matrix}3\\1\\-2\end{matrix}+\lambda\begin{pmatrix}2\\-1\\2\end{pmatrix}\right]-\begin{pmatrix}3\\1\\7\end{pmatrix}=\begin{pmatrix}0 & + & 2\lambda\\ 0 & - & -\lambda\\ -9 & + & 2\lambda\end{pmatrix}$ $\underset{\sim}{PR}\bullet\underset{\sim}{U}=\begin{pmatrix}0 & + & 2\lambda\\ 0 & - & \lambda\\ -9 & + & 2\lambda\end{pmatrix}\begin{pmatrix}2\\-1\\2\end{pmatrix}=0$ $4\lambda+\lambda-18+4\lambda=0$ $\lambda=2$ $\underset{\sim}{PR}=\begin{pmatrix}4\\-2\\-5\end{pmatrix}\Rightarrow D=\lvert\underset{\sim}{PR}\rvert=\sqrt{16+4+25}=6.7082$ $\sqrt{45}=6.7082$	B_1 for $\underset{\sim}{PR}$ M1(for dotting) A_1(for λ=2) M_1 (for both $\underset{\sim}{PR}$ and getting it's magnitude) A_1 = (for distance from P to the line)
8.	 $\frac{R}{H}=\tan 30^0$ $\frac{r}{h}=\frac{R}{H}$ $\frac{r}{h}\tan 30^0$ $r=\frac{h}{3}$	B_1
	$V=\pi\left(\frac{h}{13}\right)^2 h=\frac{\pi}{3}h^3$ $\frac{dv}{dh}=\pi h^2$ but $\frac{\pi h^3}{3}=9\pi$ h=~~3~~ 4.33 $\frac{dh}{dt}=\frac{dv}{dt}\div\frac{dv}{dh}$	$M_1$for $\frac{dv}{dh}$ B_1 for h = 3 when volume left is $9\pi cm^3$ $M_1$for substituting for $\frac{dv}{dh}$ and h

	$= -9 \times \frac{1}{\pi h^2}$ $\frac{dy}{dt} = \frac{-9 \times 3}{\pi h^2}$ $= -9 \times \frac{1}{\pi (3)^2}$ $\frac{dy}{dt} = \frac{-27}{\pi (4.33)^2} =$ .4591 cm/min $= \frac{-1}{11}$ or 0.3183*cm* per minute Hence it is decreasing at rate of 0.3183 cm per minute.	A₁ for rate.
		05
9.	(a) Let $Z_1 = 2 - i$, $Z_2 = 3i - 1$ and $Z_3 = (i+3) = -3-i$ (i) $\lvert Z_1 \rvert = \sqrt{2^2 + (-1)} = \sqrt{5} = \lvert Z_1 \rvert^2 = \left(\sqrt{5}^2\right) = 5$ $\lvert Z_2 \rvert = \sqrt{3^2 + (-1)^2} = \sqrt{10}$ $\lvert Z_3 \rvert = \sqrt{1^2 + 3^2} = \sqrt{10} = \lvert z_3 \rvert^3 =$ ~~$\sqrt[10]{10}$~~ $10\sqrt{10}$ $\left(Z^2{}_1\right) =$ ~~666.86^0~~ $\sqrt{5}^2 \times \sqrt{10} = \sqrt{10}^3 = \frac{1}{2}$ (ii) Arg $Z_1 = \tan^{-1}\left(\frac{1}{2}\right)$  $360^0 \tan^{-1}\left(\frac{1}{2}\right)$ 333.43^0  Arg $\left(Z^2{}_1\right) = 666.86^0$ Arg $(z) = 0^0$ $Arg(Z_2) = \tan^{-1}\left(\frac{3}{-1}\right)$ Hence $z = r$ $= 180^0 - \tan^{-1}(3)$ $z = 0.5(\cos\theta + i\sin\theta)$ $= 108.43^0$ $Arg(Z_3) = \tan^{-1}\left(\frac{-1}{-3}\right) = 18.43$ ~~$+180^0 = 198.43$~~ 3×18.43 $Arg(Z_3^3) = 198.43 \times 3 = 595.29^0$ 55.29 Arg Z $= 666.86^0 + 108.43^0$ ~~-595.29^0~~ 55.29 ~~$= 180^0$~~ 720^0 Z= $\frac{1}{2}(\cos 180 + \sin 180^0)$ $\frac{1}{2}$ cos0 + i sin 0	M₁ method A₁ for modulus of Z. M₁ for method A₁(argument of Z) For substitution and correct polar form of Z M, A
		06
	OR $Z = \frac{(2-i)^2(+3i-1)}{\left[-(i+3)^3\right]}$ $= \frac{\left[(2-i)(2-i)\right](+3i-1)}{-(i+3)\left[(i+3)(i+3)\right]}$	M₁

$$= \frac{\left(4-2i-2i+i^2\right)(3i-1)}{-(i+3)\left(i^2+3i+3i+9\right)}$$

$$\frac{(4-1-4i)(3i-1)}{-(i+3)(-1+6i+9)}$$

$$\frac{(3-4i)(3i-1)}{-(i+3)(8+6i)}$$

$$= \frac{9i-3-12l^2+4i}{-\left(8i+6i^2+24+18i\right)} = \frac{9+13i}{(26i+18)} = \frac{-9+13i}{(18+26i)}$$

$$\frac{(9i-13i)1(8-26l)+4i}{(18+26i18+26i)} = \frac{162-234i+234i+338}{324-468i+468i+676} = \frac{-500}{1000} = \frac{-1}{2}$$

A₁

$$|Z| = \sqrt{\left(\frac{-1}{2}\right)^2 + (0)^2} = \tfrac{1}{2}$$

$= \frac{1}{2}$

~~B₁~~ M1

$$ArgZ = \tan^{-1}\left(\frac{0}{\frac{-1}{2}}\right) = 180^0 - \tan^{-1}\left(\frac{0}{\frac{1}{2}}\right)$$

$= 0$

$180^0 - 0^0$

$= 180^0$

~~B₁~~ A1

$$Z = \frac{1}{2}\left(\cos 180^0 + i\sin 180^0\right)$$

$\frac{1}{2}\left(\cos 0^0 + i \sin 0\right)$

M₁A₁

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9. (b)

Let Z = $x+iy$

$z-2+i = x+iy-2+i$

$|z-2+i| = \sqrt{(x-2)^2+(y+1)^2}$

$|z-2+i|^2 = (x-2)^2+(y+1)^2$

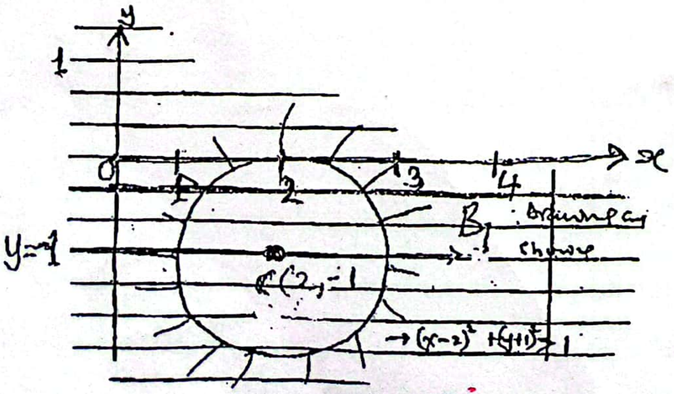
B₁ finding the models of complex

B₁ Squaring both side.

Is the equation of the circle of centre (2, 1) and $\sqrt{(x-2)^2+(y+1)^2} = 1$ means the radius is 1 unit.

M1 writing the centre and ~~of~~ radius.

A₁ CAO

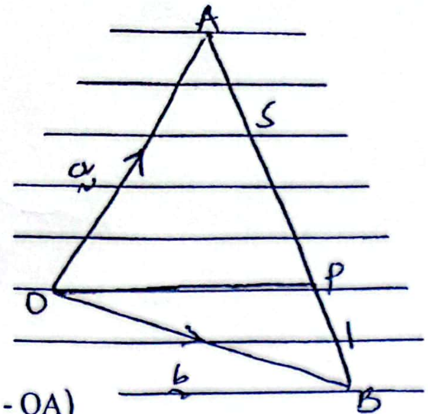
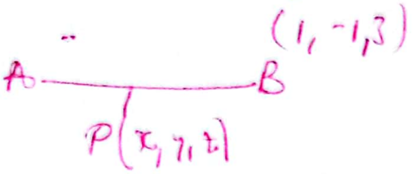


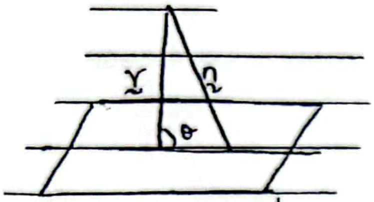
B₁ Drawing and showing

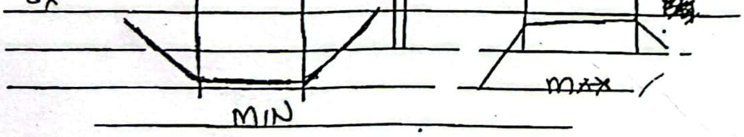
B₁ for centre of wanted region.

Complex number of it's centre is Z-i

06

(a) 	
$\underset{\sim}{OP} = \underset{\sim}{OA} + \underset{\sim}{AP}$	
$OP = \underset{\sim}{a} + \frac{5}{6}(OB - OA)$ 	
$= \underset{\sim}{a} + \frac{5}{6}(\underset{\sim}{b} - \underset{\sim}{a})$	M$_1$ finding vector OP in terms of a and b
$= \underset{\sim}{a} + \frac{5\underset{\sim}{b}}{6} - \frac{5}{6}\underset{\sim}{a}$ $x = \frac{5(1)+1(1)}{6} = 1$	
$= \frac{\underset{\sim}{a} + 5\underset{\sim}{b}}{6}$	A$_1$ C.A.O
$y = \frac{5(-1)+1(0)}{6} = -5/6$	
$\underset{\sim}{OP} = \frac{1}{6}[\,i + k + 5(i - j + 3k)]$ $z = \frac{5(3)+1(1)}{6} = 8/3$	
$OP = \frac{1}{6}[i + k + 5i + 15k]$ $op = i - 5j/6 + 8/3 k$	M$_1$ Substituting for a and b
$OP = \frac{1}{6}[6i - 5j + 16k]$	A$_1$ C.A.O
Therefore the position vector of P is $\frac{1}{6}(6i - 5j + 16k)$	
(b)(i) $n = \begin{vmatrix} 1 & -3 & 3 \\ -1 & -3 & 2 \end{vmatrix}$	
$n = i\begin{vmatrix} -3 & 3 \\ -3 & 2 \end{vmatrix} - j\begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix} + k\begin{vmatrix} 1 & -3 \\ -1 & -3 \end{vmatrix}$	M$_1$
$n = i(^{-}9 + 6) - j(^{-}3 - 2) + k(3+3)$ $n = i\,(6+3) - j\,(2+1) + k(-3-3)$ $n = {}^{-}3\underset{\sim}{i} + 5\underset{\sim}{j} + 6\underset{\sim}{k}$ $\quad n = -i - j - 2k$	A$_1$ Finding the normal vector
Hence vector normal to the plane is $n = -3i + 5j + 6k$ Suppose the plane contains any point Q (x, y, z)	
Using $\underset{\sim}{r} \bullet \underset{\sim}{n} = \underset{\sim}{n} \bullet \underset{\sim}{a}$ $\quad r.n = n-a$	
$\begin{pmatrix} x \\ y \\ z \end{pmatrix} . \begin{pmatrix} -3 \\ 5 \\ 6 \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \\ 6 \end{pmatrix} . \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ $\quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$	M$_1$ Applying the dot product

	$-3x+5y+6z=-3-10+18$ $-3x+5+6z=5$ $x+y+2z=5$ OR $3x-5y-6z+5=0$	A_1 for equation of the plan
		04
	(b) (ii) Let r_1 = vector parallel to the plane $r_1 = 4i + 3j + 2k$ Plane ~~$-3x+5y+6Z=0$~~ $x+y+2z=5$  $r.n=\lvert r\rvert\lvert n\rvert\sin\theta\rvert$ $\theta=\sin^{-1}\left[\frac{r.n}{\lvert r\rvert\lvert n\rvert}\right\rvert$ $\theta=\sin^{-1}\left[\frac{\begin{pmatrix}4\\3\\2\end{pmatrix}.\begin{pmatrix}-\overset{1}{\cancel{3}}\\\overset{1}{\cancel{5}}\\\overset{2}{\cancel{6}}\end{pmatrix}}{\sqrt{4^2+3^2+2^2},\sqrt{(-3)^2+5^2+6^2}}\right]$ $\theta=\sin^{-1}\left[\frac{15}{\sqrt{29}.\sqrt{70}}\right]$ ~~$\theta=19.44625999$~~ $\theta = 56.5°$ $\therefore$ The angle the line makes with the plane is 19.4463^{0}	B_1 stating the dot product M_1 substituting M_1 Finding the value of θ A_1 CAO.
		04
11	**(a)** $x=\frac{t}{1+t}, y=\frac{t^2}{1+t}$ $x+xt=t$ $t(1-x)=x$ $t=\frac{x}{1-x}$ $y=\frac{\left(\frac{x}{1-x}\right)^2}{1+\frac{x}{1-x}}$	M_1 Substituting for t in y.

<table>
<tr><td></td><td>$$y=\frac{\frac{x^2}{(1-x)^2}}{\frac{1-x+x}{1-x}}$$
$$y=\frac{x^2}{(1-x)^2}\times\frac{1-x}{1}$$
$$y=\frac{x^2}{1-x}$$</td><td>A_1 CAO</td></tr>
<tr><td></td><td>(b)
$$y=\frac{x^2}{1-x}$$
$$\frac{dy}{dx}=\frac{(1-x)2x-x^2(-1)}{(1-x)^2}$$
$$\frac{dy}{dx}=\frac{2x-x^2}{(1-x)^2}$$
At turning point $\frac{dy}{dx}=0$
$$\frac{2x-x^2}{(1-x)^2}=0$$
$x=0$ or $x=2$
When x = 0 y = 0 (0, 0)
When x = 2 y = - 4 (2, -4)
<table>
<tr><td></td><td>L</td><td>x=0</td><td>R</td><td>L</td><td>x= 2</td><td>R</td></tr>
<tr><td>$\frac{dy}{dx}$</td><td>-ve</td><td>0</td><td>+ve</td><td>+ve</td><td>0</td><td>-ve</td></tr>
</table>


= (0, 0) minimum turning point.
= (2, -4) Maximum turning point.</td><td>M_1 for differentiating and (correct derivative)

A_1

B_1 getting turning points

M_1 (for testing the nature)

For both minimum turning point
A_1 maximum turning point</td></tr>
<tr><td></td><td>(c)
Vertical asymptote fix is not defined.
1- x = 0
x = 1</td><td>B_1 for vertical asymptote</td></tr>
</table>

<table>
<tr>
<td></td>
<td>
Slanting asymptotes
$$\begin{array}{r} -x-1 \\ 1-x\overline{)\,x^2} \\ \underline{-x^2-x} \\ x \\ \underline{-x-1} \\ 1 \end{array}$$
$y=-x-1+\dfrac{1}{1-x}$
$As\ x\to\pm\propto\ \dfrac{1}{1-x}\to 0$
$y=-x-1$
Is the stating asymptote
Critical values are 0, 1
<table>
<tr><th></th><th>X < 0</th><th>0 < X < 1</th><th>X > 1</th></tr>
<tr><td>X^2</td><td>+</td><td>+</td><td>+</td></tr>
<tr><td>1-X</td><td>+</td><td>+</td><td>-</td></tr>
<tr><td>y</td><td>+</td><td>+</td><td>-</td></tr>
</table>
<i>Graph at the back</i>
</td>
<td>
B_1 Finding vertical Asymptote
M_1 for the table
</td>
</tr>
<tr>
<td>12</td>
<td>
(a)
$y=e^{2x}\sin 3x$
$y=e^{2x}\sin 3x$
$\dfrac{dy}{dx}=2e^{2x}\sin 3x+3e^{2x}\cos 3x$
$\dfrac{dy}{dx}=2y+3e^{2x}\cos 3x$
$3e^{2x}\cos 3x=\dfrac{dy}{dx}-2y$
$\dfrac{d^2y}{dx^2}=2\dfrac{dy}{dx}+6e^{2x}\cos 3x-9e^{2x}\sin 3x$
$\dfrac{d^2y}{dx^2}=\dfrac{2dy}{dx}+2\left(3e^{2x}\cos 3x\right)-9y$
$\dfrac{d^2y}{dx^2}=\dfrac{2dy}{dx}+2\left(\dfrac{dy-2y}{dx}\right)-9y$
$\dfrac{d^2y}{dx^2}=\dfrac{2dy}{dx}+\dfrac{2dy}{dx}-4y-9y$
$\dfrac{d^2y}{dx^2}=\dfrac{4dy}{dx}-13y$
$\dfrac{d^2y}{dx^2}=\dfrac{4dy}{dx}+13y=0$
</td>
<td>
M_1 (for 1st derivative)
B_1 (for making $3e^{2x}\cos 3x$ a subject)
M_1 (for 2nd derivative)
M_1 (for substituting $3e^{2x}\cos 3x$)
A_1 (for $\dfrac{d^2y}{dx^2}$)
B_1 (for as required)
</td>
</tr>
</table>

	(b) $\int_0^{\frac{3}{2}}\frac{x^2}{\sqrt{1-x^2}}dx$ Let $1-x^2=1-\sin^2 u$ $x=\sin u$ $dx=\cos u du$	B_1 Differentiate x with ……
	$\begin{array}{ccc} x & \sin u & u \\ 0 & 0 & 0 \\ \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\pi}{3} \end{array}$	B_1 Changing the limits
	$\int_0^{\frac{\pi}{3}}\frac{\sin^3 u}{\cos u}\cos u du$	M_1 substitutes for dX and changing limits.
	$\int_0^{\frac{\pi}{3}}\sin^3 u du$ $\int_0^{\frac{\pi}{3}}\sin u\left(1-\cos^2 u\right)du$ $\int_0^{\frac{\pi}{3}}\left(\sin u-\sin u\cos^2 u\right)du$ $\left\vert -\cos u+\frac{1}{3}\cos^3 u\right\vert_o^{\frac{\pi}{3}}$	M_1 Finding the integral
	$\left(-\cos\frac{\pi}{3}+\frac{1}{3}\cos^3\frac{\pi}{3}\right)$-$\left(-\cos+\frac{1}{3}\cos^3 o\right)$	M_1 Substituting the new Limits
	$=\frac{-1}{2}+\frac{1}{3}\left(\frac{1}{2}\right)^3-\left(-1+\frac{1}{3}\right)$ $==\frac{-1}{2}+\frac{1}{24}+1-\frac{1}{3}$ $= 5/24$ $=0.208333$	A_1 CAO
		12
13	(a) L.H S Sin $(2\sin^{-1}x+\cos^{-1}x)$ Let $A=\sin^{-1}x$; $\sin A=x$ $B=\cos^{-1}x$; Cos B $=x$	B_1 Expressing both sin A and cos B in terms of x
	Sin (2A +B) = Sin 2Acos B + sin B cos 2A = 2sinAcosAcos B + sin B(1-2sin^{2}A)	B_1 Expanding the identity

	But sin B = $\sqrt{1-\cos^2 B}$ $= \sqrt{1-x^2}$ Cos A = $\sqrt{1-\sin^2 A}$ $= \sqrt{1-x^2}$	For both expressing sin Cos A interms x B_1
	$\Rightarrow \text{Sin}\ (2A+B) = 2\sin A\cos A\cos B + \sin B(1\text{-}2\sin^2 A)$ $= 2x\left(\sqrt{1-x^2}\right)x + \left(\sqrt{1-x^2}\right)\left(1-2x^2\right)$ $= 2x^2\sqrt{1-x^2} + \sqrt{1-x^2}\left(1-2x^2\right)$	M_1Substituting in the formular
	$= \sqrt{1-x^2}\left(2x^2+1-2x^2\right)$ $= \sqrt{1-x^2}$ $\therefore \sin\left(2\sin^{-1}x + \cos^{-1}x\right) = \sqrt{1-x^2}$	A_1 CAO
		05
13	**(b)** $Sin3\theta = \sin(2\theta+\theta)$ $= \sin 2\theta\cos\theta + \sin\theta\cos 2\theta$	M_1 Expanding compound angle
	$= 2\sin\theta\cos^2\theta + \sin\theta(1-2\sin^2\theta)$ $= 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta$	M_1 change to single angle
	$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$ $= 3\sin\theta - 4\sin^3\theta$	$A_1 =$ correct expansion
	Hence From sin3θ = 3sinθ -4sin^{3}θ 1 = 6t – 8t^3 1=2(3t – 4t^3) t = sin θ 1 = 2(3sinθ -4sin^{3}θ) 1 = 2sin3θ Sin3θ = $\frac{1}{2}$	B_1 solving for 3θ
	3θ =sin$^{-1}\left(\frac{1}{2}\right)$.	M_1 for reading angles
	3θ=30^0, 150^0 , 390^0, 510^0, 750^0, 870^0 θ=10^0, 50^0, 130^0, 170^0, 250^0, 290^0 t = sin 10^0, sin 50^0, sin 130^0, sin 170^0, sin 250^0, sin 290^0	A_1 correct values of θ
	t = 0.1736, 0.7660, 0.1736, -0.9397, -0.9397. ∴ t = 0.1736, 0.7660, -0.9397.	B_1 All 3 correct values of t
		07

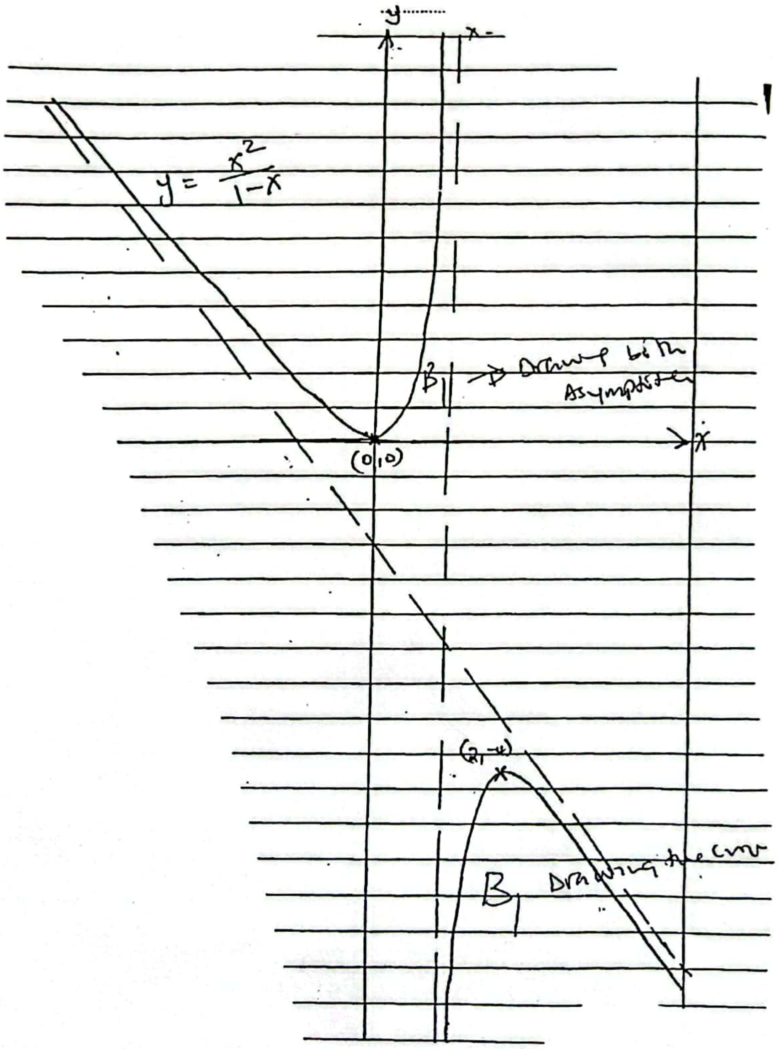
	$\frac{x^2}{a^2}+\frac{(mx+c)^2}{b^2}=1$	
	$b^2x^2+a^2m^2mcx+a^2c^2=a^2b^2$	
	$\left(b^2+a^2m^2\right)x^2+\left(2a^2mc\right)x+\left(a^2c^2-a^2b^2\right)=0$	M$_1$(for substituting for y)
	For tengency $B^2-4AC=0$	
	$=\left(2a^2+m^2\right)^2-4\left(b^2+a^2m^2\right)\left(a^2c^2-a^2b^2\right)=0$	B$_1$ (for tangency) for substitution
	$=4a^4m^2c^2-4b^2a^2c^2+4b^4a^2-4a^4m^2c+4a^4m^2b^2=0$	
	$=\frac{4b^2a^2c^2}{4a^4b^2}=\frac{4b^4a^2}{4b^2a^2}+\frac{4a^4m^2b^2}{4b^2a^2}$	M$_1$ (method)
	$c^2=b^2+a^2m^2$	A$_1$(for required equation) **04**
	(i) Given $a^2=23and\quad b^2=3$ $=c^2=3+23m^2\rightarrow(1)$ Given $a^2=14$ and $b^2=4$ $=c^2=4+14m^2\rightarrow(2)$	M$_1$(for (1) and (2) equation
	(1) - (2) $C^2 = 3 + 23m^2$ $\underline{C^2 = 4 + 14m^2}$ - $0 = -1 + 9m^2$	
	$m=\pm\frac{1}{3}$	A$_1$ (for M = $\pm {}^1/_3$)
	$c^2=4+14\left(\frac{1}{9}\right)$ $9c^2=36+14$ $9c^2=50$ $c=\frac{\pm\sqrt{50}}{3}$	B$_1$ (for C = $\pm \subseteq 2$) $\sqrt{\ }/3$
	$y=\pm\frac{1}{3}x\pm\frac{\sqrt{50}}{3}$ or $3y=\pm x\pm\sqrt{50}$ or $3y=\pm x\pm5\sqrt{2}$	B$_1$ for correct eqn
		04
14	(ii) Given $a^2=16$ and $b^2=9$ but $c^2= b^2 + a + 2m^2$ $c^2 = 9 + 16m^2$ ……….(1)	
	From $y = mx + c$ $3= -3m + c$ $c = 3\ (1+m)$…………(2)	M$_1$ (for both (1) and (2)
	$\left[3(1+m)\right]^2=9+16m^2$ $9\left(1+2m+m^2\right)=9+16m^2$ $9+18m+9^2=9+16m^2$ $18m=7m^2$	M$_1$ (for substituting for c in equation (1)from equation (2))

	$(7m-18)m=0$ $m=0, \frac{18}{7}$ When m = 0 = c = 3(1) = 3 y = 3 is equation of tangent at (-3, 3) When m = $\frac{18}{7}$ = c = 3(1 + $\frac{18}{7}$)= $\frac{75}{7}$ y = $\frac{18}{7}x+\frac{75}{7} or 7y=18x+75$ is other equation of tangent at (-3, 3)	A$_1$(for both m = 0 and m = B$_1$(for both equations of tangents at (-3, 3)
		04
15	**(a)** Let the first term be a the common difference be d nth term of AP = a+ (n-1)d a + 7d = 2(a + 2d) …………………(1) sum of n term is given by Sn = $\frac{n}{2}$ (2a + (n - 1)d) $s_8=39=\frac{8}{2}(2a+7d)$..............(2) From equation (1) a + 7d = 2a + 4d a = 3d ……………..⊗ Substituting equation ⊗ in equation (ii) $\frac{8}{2}(2(3d)+7d)=3a$ $4(6d+7d)=3a$ $52d=3a$ $d=\frac{3}{4}$........⊗⊗ Substituting equation ⊗⊗ into ⊗ a = 3 ($\frac{3}{4}$) a = $\frac{9}{4}$ The first three term of the progression are given by a, a + d, at 2d $\frac{9}{4}$, 3, $\frac{15}{4}$ = Sum of n term is given by $sn=\frac{n}{2}(2a+(n-1)d)$ $sn=\frac{n}{2}\left(2\left(\frac{9}{4}\right)+(n-1)\frac{3}{4}\right)$	M$_1$ for the sum of eight terms B$_1$ C.A.O B$_1$ C.A.O B$_1$ C.A.O M$_1$ for substitution in the formula of sum of nth term

	But $S\alpha - Sn\angle 10^{-6}$ $\frac{5}{4}-\frac{5}{4}\left(1-\left(\frac{1}{5}\right)^{n}\right)\angle 10^{-6}$ $\frac{5}{4}\left(\frac{1}{5}\right)^{n}\angle 10^{-6}$	B₁(Difference between $S_{\infty}-S_{n}$)
	$5\left(\frac{1}{5}\right)^{n}\angle 10^{-6}\times 4$ $5^{1-n}\angle 4\times 10^{-6}$ Introducing $\log_{10}$	
	$\log 5^{1-n} < \log\left(4x10^{-6}\right)$ $1-n < \frac{\log\left(4\times 10^{-6}\right)}{\log 5}$	M₁ for introducing $\log_{10}$ or In.
	$1-n < -7.7227$ $n > 1+7.7227$ $n > 8.7227$ $\therefore n = 9$ *terms*	A₁ (correct value of n)
		06
16	(a)	
	x = number of antelopes present $\frac{dx}{dt} \propto (x+5)$	M₁ writing differential equation
	$\frac{dx}{dt} = -kdt$ $(-5+x)$	M₁ introducing the scalar k.
	$\int\frac{dx}{(5+x)} = \int -kdt$	M₁(for separating variables)
	$In(5+x) = -kt + c$	M₁(integrating on both sides).
	When t = 0 x = 60	A₁ correct integral.
	ln (5 + 60) = -k(0) + c	M₁ substitute for t and x
	ln (65) = c	A₁ correct expression of c.
	ln (5+ x) = -kt + ln 65 ln (5 + x) –ln (65) = -kt $ln\left(\frac{5+x}{65}\right) = -kt$ $5+x = 65e^{-kt}$ $x = \left(65e^{-kt}\right)-5$	

	$Sn=\frac{n}{2}\left(\frac{18}{4}+\frac{3}{4}n-\frac{3}{4}\right)$ $=\frac{n}{2}\left(\frac{15}{4}+\frac{3}{4}n\right)$ $\therefore Sn=\frac{3}{8}n(n+5)$	A_1 C.A.O
		06
15	**(b)** The series $1+\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+$.......... is a G.P with 1st term a = 1 common ratio r = $\frac{1}{5}$ Sum ~~=~~ to infinity $S\alpha=\frac{a}{1-r}$ $=\frac{1}{1-\frac{1}{5}}$ $=\frac{1}{\frac{4}{5}}$ $S\infty=\frac{5}{4}$ Let the number of term whose sum will differ from sum to infinity by less that 10^{-6} ~~then.~~ be n $Sn=\frac{a(1-r^n)}{1-r}$ $\lvert r\rvert\angle 1$ $a=1$ $r=\frac{1}{5}$ $\frac{1\left(1-\left(\frac{1}{5}\right)^n\right)}{1-\frac{1}{5}}$ $=\frac{1-\left(\frac{1}{5}\right)^n}{\frac{4}{5}}$ $=\frac{5}{4}\left(1-\left(\frac{1}{5}\right)^n\right)$	M_1 for substitution A_1 (Getting the sum to infinity) B_1 (getting the sum of n - terms)

	When t = 6 months, $x = 40$ $40 = 65e^{-6k} - 5$ $45 = 65e^{-6k}$ $\frac{9}{13} = e^{-6k}$ $k = \frac{1}{6}ln\left(\frac{9}{13}\right)$ $\frac{1}{6}\ln\left(\frac{13}{9}\right)$ $x = \frac{1}{6}e^{\frac{1}{6}ln}\left(\frac{9}{13}\right) - 5$	B$_1$ (for value of k) A$_1$ for correct solution of D.E
		09
	(b) On 15th November 2019, t = 8.5 $x = 65e^{\frac{8.5}{6}ln}\left(\frac{9}{13}\right) - 5$ $x = 33.6$ antelopes $x = 33$ antelopes.	B$_1$ for t = 8.5 M$_1$ substituting t = 8.5 A$_1$ CAO
		03



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